More Gedanken Calculus including Definite Integrals

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Tip: Live to learn and learn to live

You are strongly recommended to browse through and use as much of How to Solve it by George Polya as possible

1) Prove that
$\frac{d^n}{dx^n}\{\frac{1}{x(x^2+1)}\} = (-1)^n n![x^{-n-1} - \cos\{(n+1)\cot^{-1}x\}(x^2+1)^{\frac{-(n+1)}{2}}]$

2) If $u_n = \int_0^{\pi/2} x^n \sin x dx$, prove that $u_n + n(n-1)u_{n-2} = n(\pi/2)^{n-1}$ and hence that $u_3 = \frac{3\pi^2}{4} - 6$

3) Prove that the area of the loop of the curve $r\cos\theta = a\cos 2\theta$ is $(1/2)(4-\pi)a^2$ and that the volume generated by the revolution of the loop about the initial line is $2(\ln 2 - 2/3)\pi a^2$.

4) If every root of the equation $f^{'}(x) = 0$ be subtracted from every root of the equation $f(x) = 0$, show that the sum of the reciprocals of the differences is zero, provided the roots of $f(x) = 0$ are all different.

5) Obtain the indefinite integral $\int \frac{dx}{\sin x(1+\sin x+\cos x)}$ in the form
$(1/2)(\tan x/2 + \log\frac{\sin x}{1+\sin x})$

7) Show that the integral
$\int \frac{(x-a)^2}{\sqrt{(x-b)(x-c)}}$
can be rationalized by the substitution $y^2 = \frac{x-c}{x-b}$
and, hence, evaluate the integral in the case $a < b < c$.

8) If r be a focal radius vector of any point on an ellipse, p the perpendicular from the focus on the tangent at the point, p the perpendicular from the focus on the tangent at the point, and 2l the latus-rectum of the ellipse, prove that the value of the integral $\int \frac{pds}{r^3}$ taken round the ellipse is $2\pi/l$.

$\smile\ \smile\ \smile$ GOOD LUCK $\smile\ \smile\ \smile$